Emissions Trading:

The Evolution of a Market for Pollution Permits

November 1999

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Summary

There is an intense debate about how best to control emissions of the gases which contribute to the greenhouse effect. The UK has signed up to a target for reductions in these emissions and a number of measures are open to the government in meeting these targets. One method of control is to allocate permits without which emissions cannot be made. Since this is a very restrictive method, one way of making it more flexible is to allow for a market in such permits.

However, there are concerns about whether such a market is possible and the UK government is currently suggesting that any scheme should be voluntary. Other governments are also investigating the potential for trading. The worry is that if only a few firms are involved, or permits are misallocated, the market may be unstable or trading may be extremely thin. This report looks at whether these worries are well founded or not.

In the report, we set out an initial framework of analysis for understanding how well a market for permits might function. In particular, we address the questions:

- how stable will such a market be?
- how many firms trading at what sort of level appear to be necessary for stability?
- what sort of prices will be established?

It is not possible to study the evolution of an existing market for permits in the UK or anywhere else, for the obvious reason that such markets do not exist. Trading does take place in markets for power and some of the power brokers offer options for dealing in emissions. However, these markets are rather different from the market in a product which has yet to exist. In the case of power trading, there are real markets, while existing emissions trading is for a product which has yet to have any real standing.

To cope with this, we investigate the problem by creating an artifical economy within a computer, populated by firms following straightforward rules of behaviour. Many different 'histories' of how firms learn to operate once a permit market is created can be generated, and the evolution of the market examined.

Of necessity, any model must be a considerable simplification of reality. This is no exception. We assume that firms are very familiar with their own businesses, and with the costs of abating pollution. However, we assume, deliberately, that firms are initially almost completely ignorant of how to behave in the permit market. They use very simple learning rules as the market develops. In this way, we can focus entirely on how the permit market develops.

Because the model is very simple, it can be solved analytically for the 'equilibrium' price of permits. At this price, the permit market can be said to be generating an efficient distribution of output and pollution.

Initially, we assume that firms are very similar to each other. They therefore have little incentive to trade. They also follow simple rules of learning about the value of permits

Under these assumptions, prices in the permit market are at first very volatile, as firms learn how to operate in the market. But, in general, even though the number of trades carried out is small, the market is an orderly one :

- price volatility in the permit market soon drops dramatically
- the market price for permits then moves around close to its theoretical equilibrium level

In other words, even in a market which by construction is 'thin', the permit price shows very considerable stability, and moves around the theoretically 'efficient' price (the price, like any market in reality, never settles for ever at a single level, but changes from period to period).

These results hold even when as few as six firms are postulated to take part in the market.

We also develop a more realistic version of the model, in which firms differ in terms of size, of the amount of pollution per unit of output, and of the costs they face in carrying out abatement of pollution.

A crucial finding is that the greater the number of ways in which firms differ, the greater the number of trades which are carried out in the permits market.

There are a number of ways in which the model can be developed still further. The learning rules, for example, can be made more sophisticated. The impact of various types of taxes and/or regulations can be incorporated into the model, and so on. But the prototype system reported here generates some very interesting findings.

Introduction

Not so long ago, the production of the products now known as greenhouse gases came free. While environmentalists had voiced concern for years about their effect on atmosphere and consequent rises in temperature, reactions to these warnings were piecemeal and halfhearted. There was no way to put a value (or disvalue) on the production of such gases. The ***Kyoto Protocol,*** agreed on 11th December 1997, changed all that. It was devised as a route to controlling global warming by setting targets for production of these commodities – though it has yet to be ratified by the US. In setting such targets, for the first time there is a way of putting a cost on these gases.

Under the agreement, the European Union is committed to reducing emissions of greenhouse gases (six gases, including carbon dioxide) to 8% below 1990 levels by the period 2008 to 2012. As its share of the EU commitment, the UK has accepted – in June 1998 – a legal target of a 12.5% reduction on a 1990 base. This means finding an additional reduction of 5 million tonnes of carbon above that which is expected to be delivered by existing policies. The government has also suggested a more exacting domestic target of a 20% reduction in carbon dioxide (CO_2) by 2010, though this is still being discussed.

The Kyoto protocol itself proposes the establishment of an international trading scheme as well as two mechanisms whereby countries can use savings elsewhere to compensate for their own emissions. The international trading scheme is still in its infancy and is not expected to be sorted out until the 6th Conference of Parties at the end of next year. Even then, issues may not be fully resolved.

Emissions Trading Proposals

In the meantime a number of countries have been working on their own schemes. Within the European Union, for example, Denmark is developing a domestic emissions trading scheme. They have introduced a tradable quota scheme for their electricity producers to begin operating from the year 2000, and targets have been set to 2003. Admittedly, the scope of the scheme is currently very limited but the intention is to expand the scheme to include other electricity producers across Scandinavia.

This could include Norway, where the Parliament has set up a commission to design a domestic emissions trading scheme. This is due to report by the end of the year. The trading system is expected to apply to those sectors which are exempt from Norway's CO_2 tax – which includes much of heavy industry - and other sectors could also be included.

In Australia, a consultation exercise is under way on a number of the difficult design issues associated with a domestic trading scheme, including allocation and coverage. Two discussion papers have already been published, and two more are in the pipeline. Although this is very much "work in progress", it seems that the intention is for as comprehensive a scheme as possible. This could involve a combined upstream and downstream approach.

The pace at which trading schemes are being developed is best illustrated by Canada and the United States, where trading in carbon is already underway. GERT – the Greenhouse Gas Emissions Reduction Trading Pilot – was established in Canada in June 1998. Since then there have been a number of trades in emissions reduction credits. These credits can be generated by reducing existing emissions; avoiding an increase in emissions; or through carbon sequestration. In the United States a shadow market in CO_2 is already up and running. This is without any rules or arrangements for carbon trading actually being in place. But there is proposed legislation in Congress which could formalise this carbon market.

In the UK, a report published by the government in November 1998 and prepared by Lord Marshall, concluded that a trading scheme offered a number of advantages in meeting emissions targets

Trading schemes give firms legal targets to reduce emissions. But they allow companies that can reduce emissions more easily to go further, and to sell the excess to companies finding it more difficult or expensive to meet their targets. In this way emissions reductions take place where it is cheapest, allowing targets overall to be reached more cost-effectively. This attractive flexibility for individual firms is combined with certainty for the regulator. With a fixed number of permits in circulation, provided that the compliance regime is robust, the regulator knows in advance what overall minimum reduction in emissions will result.[1]

[1] *"Economic Instruments and the business use of energy", A Report by Lord Marshall, November 1998, para 44, p11.*

However, despite recognising these clear advantages in principle, Marshall was sceptical about the speed at which an emissions trading system can be introduced, and about how wide its coverage can be. Nonetheless, government remains keen on developing a trading scheme as the following quotations, taken from a recent speech by Ian Coates at the DTI, make clear.

"I firmly believe that emissions trading will form a crucial element in our long-term strategy to reduce greenhouse gas emissions."

"I believe such a trading system can both encourage business find the most cost-effective ways of reducing emissions, and give invaluable practical experience of a trading system ahead of the introduction of an international trading system."

"The development of a domestic scheme offers us the opportunity to make significant progress in reducing our emissions, but in a way that allows the most cost-effective approaches to be employed across industry."

These are quotations from John Battle – then Minister of State at DTI – Patricia Hewitt – then Economic Secretary at the Treasury – and Michael Meacher – who remains the Minister for the Environment.

The desire to introduce a trading scheme is not just confined to the government. A group convened by ACBE (Advisory Committee on Business and the Environment) and CBI(Confederation of British Industry) has been working since the middle of 1998 to organise an emissions trading scheme for the UK. This effort has been accelerated by the government's proposals for the Climate Change Levy and the negotiations with energy intensive sectors to set targets for emissions reduction alongside a possible reduction in the tax rate.

A set of proposals has now been developed which are practical and workable. They were presented to ministers on October 27th and have been extensively discussed with officials of the (Department of Environment, Transport and the Regions (DETR), Department of Trade and Industry (DTI) and the Treasury. They have also been reflected in the Chancellor's Pre Budget Report on November 9th this year.

Many details remain to be worked out – but now that the principles have been set, a new worry has emerged. It is clear that it is possible to set up a market, but this simply begs the question of whether the market itself will work. If only a few companies trade, or they are ignorant of how to assess costs and benefits, the market may be far too volatile or thin.

This report looks at these problems. It uses innovative techniques to look at how a market in permits for emissions could develop, even under very restrictive conditions. It shows that firms are capable of learning how to trade, even when they have short memories and the allocation of permits has uncertainties attached to it.

This is obviously not the final word. The proposed scheme has a number of special features which we are working on incorporating into the model. These include the ability to bank permits for specified periods as well as the existence of firms with allocations on different bases – those who have accepted absolute targets compared to those with energy efficiency targets for example.

1 The structure of the model

1.1 Overview

We consider a simple situation in which firms produce one good and ‘pollution’. They have costs associated with producing their output and they can also reduce pollution by incurring costs. They have an initial allotment of pollution permits for one period which they may buy and sell before making their production and pollution level choices. They learn about the profitability of their behaviour in the permit market and use their experience to modify their choices.

In order to illustrate the process of how a market in permits evolves, we assume to begin with that all firms in the model are identical. But even under this highly simplifying assumption, as a result of the operation of the permit market, production and profit levels subsequently differ across firms. We set out the results of the evolution of both the market price for permits and the number of trades which are are carried out.

Next, we show the results when the model is made more realistic. We look at what happens when firms differ in size, in their allocation of permits, in their overall cost functions, and in their costs of reducing pollution (the abatement function). The model is set up to be as realistic as possible in terms of scales of output and pollution, using information in the Marshall Report, *Economic instruments and the business use of energy*, published in November 1998. A description of how we used this information is set out in Appendix 2.

The formal mathematical statement of the model is given in Appendix 1.

1.2 Input and Commodity Markets

We assume that firms can sell all of their output at a constant price. In other words we assume that the demand for the final product is perfectly elastic. The same applies to *input markets* (apart from the pollution permits). We make these simplifying assumptions in order to focus on the market for permits. Whilst they are unrealistic over the whole range of possible production levels of actual firms, in terms of marginal variations in output around existing levels, they are not completely unreasonable.

1.3 Technology

Each firm knows the cost of producing any amount of its commodity . It knows how much pollution is generated by any given level of production and also knows the cost of abating pollution by any amount. The firm does not know the structures of costs and abatements of other firms. Again, in terms of small movements around existing levels of production, these are not completely unrealistic assumptions.

1.4 Pollution and permits

The amount of pollution a firm can generate is limited by its holdings of pollution permits. Each single permit allows a firm to produce one unit of pollution. Permits are valid for just one period.

1.5 Time and the evolution of the model

The model is essentially a single period model. Permits are issued which are valid for a fixed amount of time. An initial sequence of trading takes place, as a result of which firms decide their output and abatement levels for the whole of the period.

In this prototype model, the steps of the sequence are somewhat artifical, in order to focus on the learning process. A similar type of sequence is used widely in economic theory, and can be thought of in the following way[2]. Each firm chooses an initial set of prices at which it would be willing to buy and sell permits. No actual trading takes place at these prices. Instead, they are submitted to a central agency or broker. This agency or broker then informs firms about what would happen to their bids, and what the price established in the market would be if trade were to take place on the basis of the initial bids.

Firms then choose what their output and abatement levels would be if trade were to take place on this basis, and compute the consequences for their profitability. They then submit a revised set of buy-and-sell prices. The agency or broker then informs them of the consequences of these new bids, and again the firms decide what output and abatement would be on the basis of these new prices, and the resulting consequences for profitability. This whole process is then repeated a large number of times.

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[2] *Economists will recognise this as being analogous to the process of tatonnement in general equilibrium theory*

The focus of the model is upon the evolution of the market price for permits during this process. Does the market price converge to its theoretical equilibrium level? If it does, how many steps of the process are required before the price gets close to this level? If the price does get close to this level, but the process of submitting bids is allowed to continue for many more steps, does the price remain close to this level?

1.6 Firm Behaviour

We assume that a firm knows its business well, and will choose the optimal output level (with corresponding optimal pollution abatement level) given its actual permit holdings. The question, then, is how a firm chooses its permit holdings. Each step, firms do the following:

- they decide a target permit level
- given their currently allocated permits, their target determines their demand or supply for permits and they then send this to the market together with a price
- if a firm would have been successful in its previous buy attempt, it will decrease its price according to a simple rule. If it would have been unsuccessful, it will increase its price, again according to a simple rule. These statements hold in reverse if the firm wants to sell.

1.7 The Market for Permits

We analyse a double auction. This lines up all bids for permits in descending order of price and all offers to sell in increasing order of price. Given these bids and offers submitted by the firms, a market clearing price is determined. This is the price at which the number of permits that individuals are prepared to sell are just equal to the number of permits that other individual firms are ready to buy. We assume that all trades would take place at that price. Since permits can only be exchanged by one unit at a time (i.e. are non-divisible) the "equilibrium price" may not exactly clear the market and some rationing might occur. This is taken care of by putting the traders in a random order, and allocating the available permits on a first-come first served basis.

1.8 Learning in the Permit Market

At the outset, firms understand very little either about the permit market, or about the value of permits. When the market for permits is introduced, each firm chooses, at random, the price at which it is prepared to buy and the price at which it is prepared to sell permits, from a uniform distribution over a specified price range which is common to all firms. In other words, firms initially value permits at random from within a specified range of values.

Learning about permits takes place according to the profits which firms would experience if actual trade were to take place at the price established during any particular step of the trading sequence. Given a firm's allocated permits and the outcome of the market for permits, it would end up with a certain permit level, which (given the production, pollution, and abatement costs) eventually determines its profits.

In other words, during the trading sequence, a firm is informed of a sequence of permits levels, one for each step of the process and the corresponding profits. We assume that firms modify their offers to buy or sell permits in the light of what happened in the previous step. If profits would have improved, they continue to change in the same direction. Slightly more formally, each firm uses a hill climbing algorithm to determine its target permit levels. That is, a firm reviews what its profits would have been over the previous k steps. It identifies the step at which its profits would have been highest, and sets its target permit level equal to its actual level at this step, plus a small random number. This rule is based upon a straightforward methodology used in search algorithms. If the number of steps, k, which the firm reviews is small, it forgets information very quickly. But even under these conditions, with k set as low as 2, the results obtained from the model are not qualitatively different to those reported here. The results set out are based upon a value of k equal to 7, which allows for a more realistic, albeit still rather naïve, learning process.

2 Numerical Analysis of the Model

2.1 Results with identical firms

In order to focus explicitly upon the evolution of the market for permits, we initially assume that all firms are identical in terms of size, the amount of pollution they produce, their overall cost functions, and their costs incurred in abating pollution.

This may be thought to be so unrealistic a set of assumptions that it is not worth exploring. But in fact it yields very valuable information. By assumption, before trading in permits is introduced, firms by definition are identical. In such circumstances, there is as little incentive to trade as possible. Yet, to anticipate, even with a very small number of firms, an orderly market in permits does develop.

The model has been simulated many times on the computer, and a large number of artifical 'histories' developed under a range of assumptions. The outcome of this very basic model is most sensitive to variations in the number of firms which are involved, and we concentrate on the impact of this in reporting the results.

It may be useful, however, to have some feel for the various scales of operation of the results of the basic model reported here. The initial range of prices at which firms value permits is set to be between 0 and 2000. Each firm receives an allocation of 100 permits in each period. The revenue from producing and selling an extra unit of output is 1000. The cost structure is such that it is never profitable for a firm to produce more than 500 units of output.

Variations in these assumptions can be made. For example, the wider the range of price from which firms form their initial views on the values of permits, the higher is the initial degree of volatility in the price of permits. The fewer permits are issued - in other words the more severe the degree of pollution control - the more volatile are the initial movements in price. But it is the number of firms to which the results of the model are most sensitive.

Figure 1 shows a typical evolution of the price of permits over the steps of the trading sequence when there are 40 firms operating in the model.

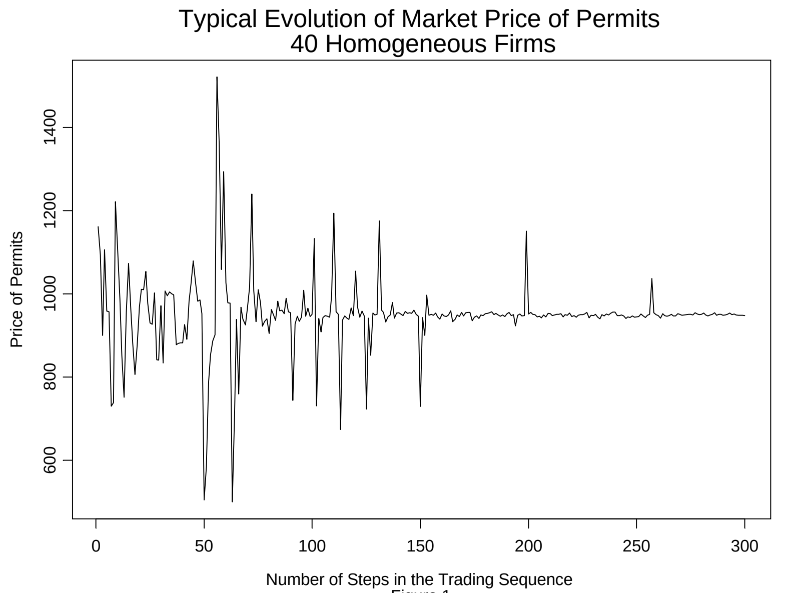

Figure 1

The chart show the qualitative properties of the evolution of price with 40 firms very clearly. There is a relatively short initial number of steps during which firms learn about the value of permits, and in which price is volatile. But the market settles rapidly to move around its theoretical equilibrium value of some 1000. The market never becomes completely static, and trading continues to take place. But even in what is, by deliberate construction, a thin market in which few permits are traded relative to the overall scale of output, price converges to around its equilibrium level. With more than 40 firms, the period of high price volatility becomes somewhat shorter, but even with 200 firms it is never eliminated. But once there are as many as 40 firms in the market, the addition of further firms makes little difference to the results.

The number of trades which would be carried out in each step of the trading process is set out for a typical solution of the model in Figure 2.

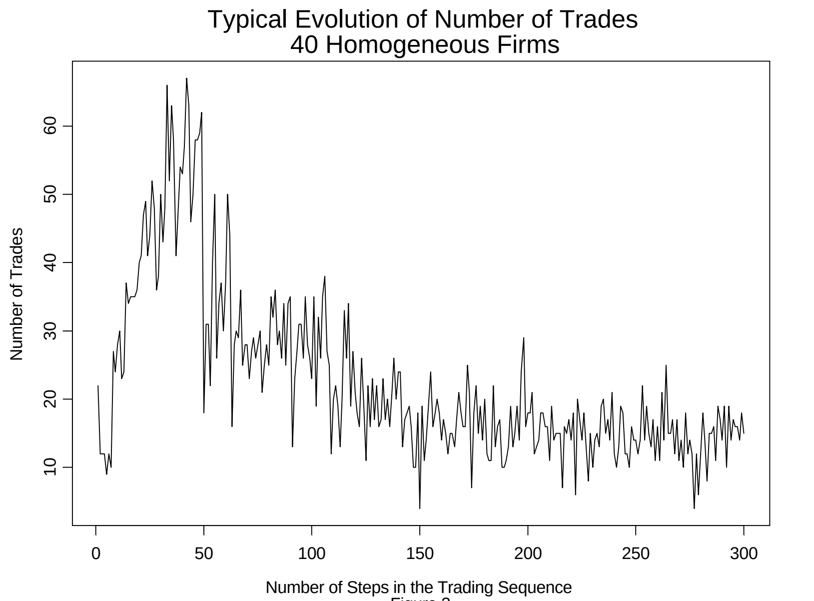

Figure 2

During an initial period of intensive learning about the permits market, the number of trades per period is of the order of 50, but then settles down to between 10 and 20.

Reducing the number of firms below 40 leads to a slight lengthening of the period of high volatility, and to a marginally higher degree of volatility even when the model has run for many steps. But convergence to equilibrium is still attained. Figure 3 illustrates a typical solution of the model with only 15 firms.

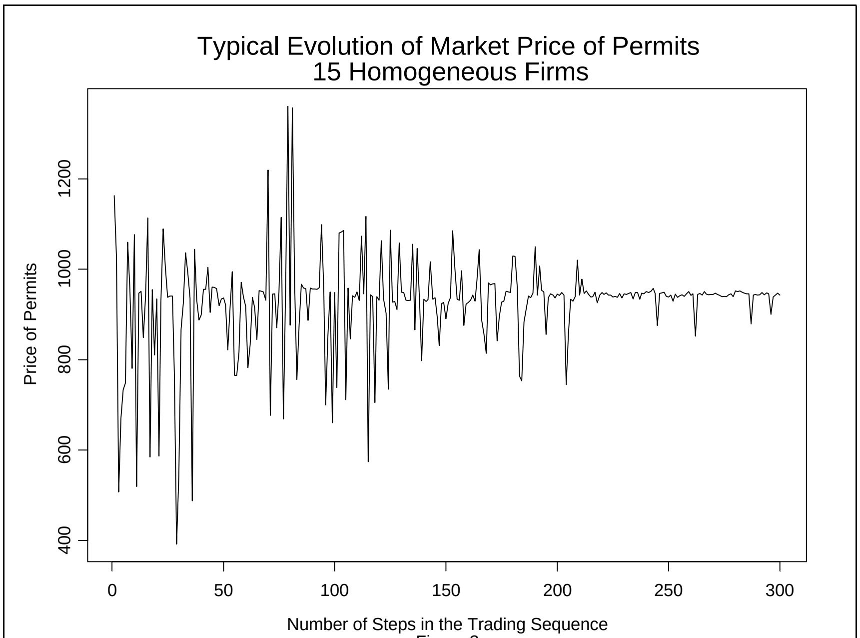

Figure 3

Comparing the results with those of figure 1, it is apparent that they are qualitatively similar. The number of trades carried out is also similar, as Figure 4 shows.

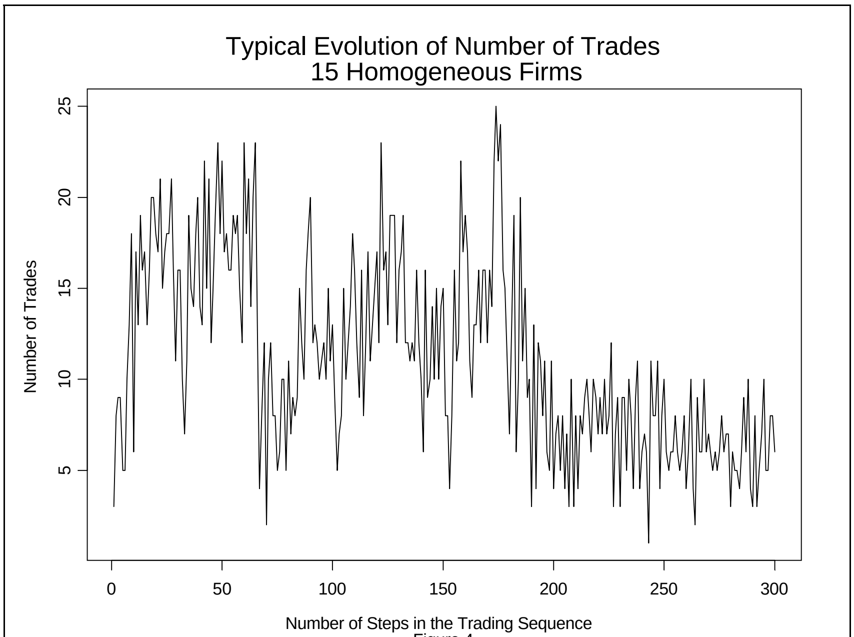

Figure 4

A different kind of behaviour is observed once the number of firms is reduced even further. A certain degree of more erratic behaviour is sometimes observed with 12 firms, and with as few as 10 firms, a typical evolution of the permit price is plotted in Figure 5.

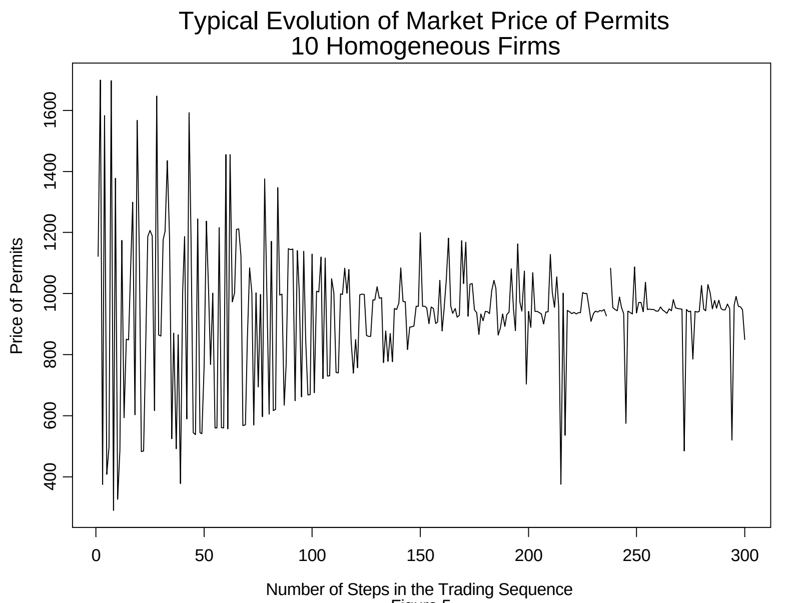

Figure 5

The price does converge around its equilibrium value, although more slowly than in the previous charts. But the relatively modest fluctuations in price which then take place are punctuated by rather more frequent, short bursts of large, erratic movements. The gaps in the solid line on the chart indicate periods when no trading takes place.

With only 10 firms, the number of trades which are carried out in a typical solution is lower than with 15 or 40 firms. This is exactly as one might expect, given the small number of firms in this example.

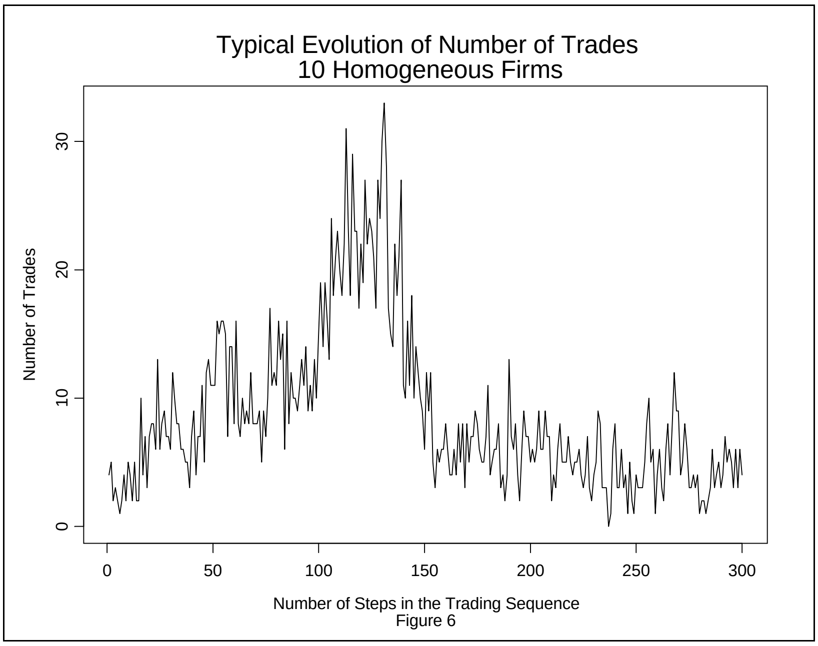

Figure 6

When the number of firms is reduced below 10, the price does still eventually converge to around its theoretical equilibrium level. But the degree of volatility can be considerably higher, and in a number of periods no trading at all takes place. It is less easy to characterise a typical solution in these circumstances, but Figure 7 sets out one example of the evolution of price when the number of firms is as few as 6. Again, the gaps in the solid line on the chart indicate periods when no trading takes place. As one would expect this occurs more frequently as the number of firms decreases.

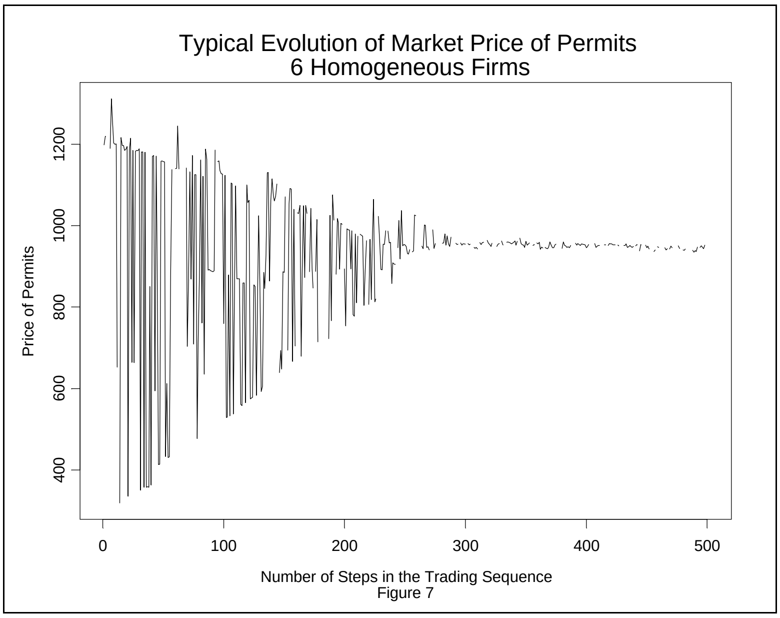

Figure 7

Figure 8 confirms the point about the number of trades when the number of firms participating in the market is very small. Even at its peak level, the number of trades in any given period is barely into double figures.

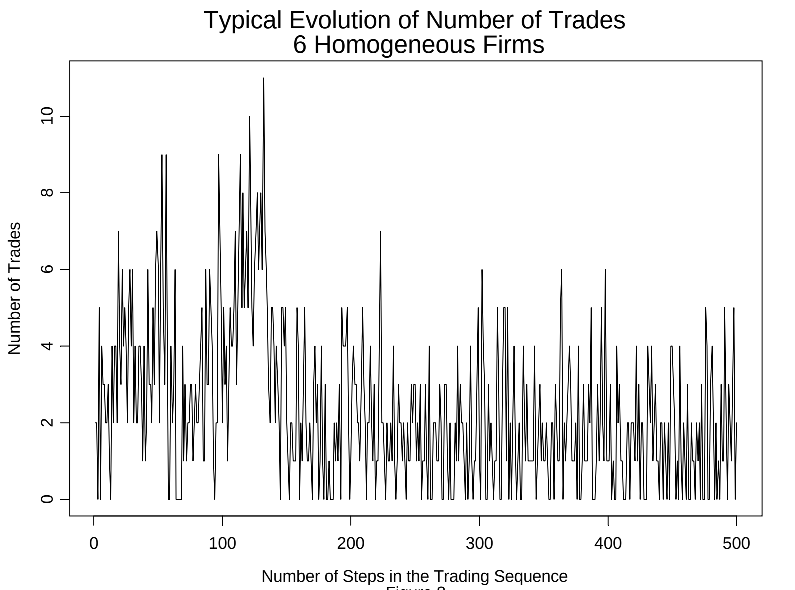

Figure 8

In summary, we have presented results in this section of the paper showing the evolution of a market for permits using a set of assumptions under which there is little incentive to trade. Deliberately, the number of firms postulated to take part in the market is very small. They are assumed to be almost wholly ignorant of the potential value of a permit before the market is introduced, and they follow a rather naive procedure for learning about the value of a permit.

But even in these circumstances, an orderly market in traded permits does develop. After a initial period of fluctuation, which tends to be larger the fewer the number of firms involved, the market settles down. Futher, it settles around its theoretical equilibrium price. This means that resources are allocated in an efficient way.

2.2 Results with more realistic firms

In this section, we report on what happens when a greater degree of realism is introduced to the nature of the firms in the model. It must be emphasised that this is not intended by any means to be an exact description of reality, either in the description of the firms, or in the particular trading scheme. The intention is still to explore at a rather general level the implications of introducing a market in which permits can be traded.

A key difference with the previous section is that firms differ in size. The largest firm is around fifteen times larger than the smallest. The degree to which firms contribute to pollution varies even more, with the heaviest polluter contributing some forty times more per unit of output than the lightest.

The Marshall report contains data on the estimated gross output of industrial sectors, and their estimated CO_2 emissions. We concentrated on the 22 industrial sectors, covering industries such as mining, paper, chemicals. Information on the number of firms in each sector is available in the August 1999 DTI Publication *SME Statistics for the United Kingdom 1998*, which despite its title contains data on firms of all sizes. We could therefore obtain the average in each industrial sector of the size of each firm and the level of pollution associated with its output.

We further assume that only one in every thousand firms takes part in the market for permits. This still gives a total of 271 firms which participate in the theoretical model - a much higher figure than the more artificial examples set out above.

Firms also differ in respect of their cost of abatement functions. The basic shape of the function is the same for all firms: the cost of reducing pollution by a single unit increases, the more the total size of the reduction. In other words, costs rise quite sharply. But the speed with which this happens is allowed to vary across firms. We do not pretend to have knowledge of the actual costs of abatment which any firm faces in reality. But real firms do differ in this respect, and our model now reflects the existence of such differences.

In Section 3.1, we reported a typical result obtained when the model is solved. In this section, we report the results of 500 repeated solutions of the model. This enables a richer view of the range of solutions to be obtained, even though the charts themselves look somewhat more complicated.

Results were obtained under three sets of assumptions about the allocation of permits:

(1) The number of permits that are allocated to each firm is exactly in proportion to their sizes, so that a firm which produces twice as much as another receives twice as many permits

(2) The same principle is used as in (1), but the actual number allocated to each firm contains a certain amount of random variation. This means that the number of permits received by 95 per cent of all firms in the model is within a range of plus or minus 20 per cent of the number they would have received purely on the basis of their relative sizes.

(3) As with (2), but the range of variation around the allocation which would have been made purely on the basis of relative size is much larger, at plus or minus 40 per cent.

With firms operating on the basis of the assumptions made above, there is far more incentive to trade in the permits market. *It is precisely the fact that firms are different which gives them more reason to trade.*

We examined this proposition thoroughly, by investigating what happens to the number of trades carried out when the assumptions of section 3.1 are relaxed both on a one-by-one basis and in various combinations. So, for example, we assumed that firms are identical except for the amount of pollution associated with each unit of output. We then restored the assumption that they are identical in this respect, but made their relative sizes different. Detailed results on this are available from Volterra Consulting. But, quite clearly, *the greater the differences between firms, the more incentive they have to trade, and the more trades are carried out.*

This can be illustrated by comparing Figures 9 and 10. Figure 9 shows the number of trades carried out in 500 solutions of the model assuming firms are identical. This is exactly as in section 3.1 above, except that the number of firms is the same as in the results reported when firms are different. In other words, rather than involving just 40, or 15 or 10 firms, Figure 9 shows the results for 271 identical firms.

The chart needs a little explanation. The middle of the three lines - the dotted line in the chart - shows the *average* number of trades carried out in each period when the model is solved 500 times. The upper line shows the *maximum* number which are carried out in each period in *any* of the 500 different solutions. In other words, it is not the outcome of the single solution which on average has the highest number of trades, but rather more than this. It shows the highest number carried out in each period in any of the individual solutions - the maximum of the maximums, as it were. The bottom line shows, similarly, the minimum of the minimums.

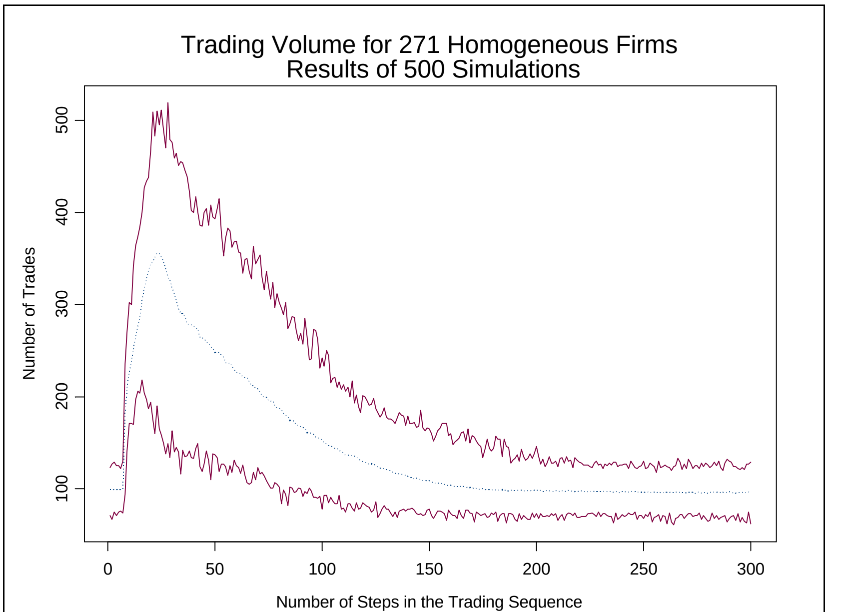

Figure 9

The number of trades carrried out is larger than with a smaller number of identical firms, but not dramatically so. Figure 2 on page x shows that with 40 firms, the number of trades per period settles down at between 15 and 20. Figure 9 shows that with approximately six times this number of firms, the number of trades is approximately six times higher on average, at around 100.

Figure 10 reports results over 500 solutions of the model for 271 firms which are different in the ways described above. The numbers of permits are allocated exactly in proportion to the relative sizes of the firms. The number of trades is substantially higher, settling around 320.

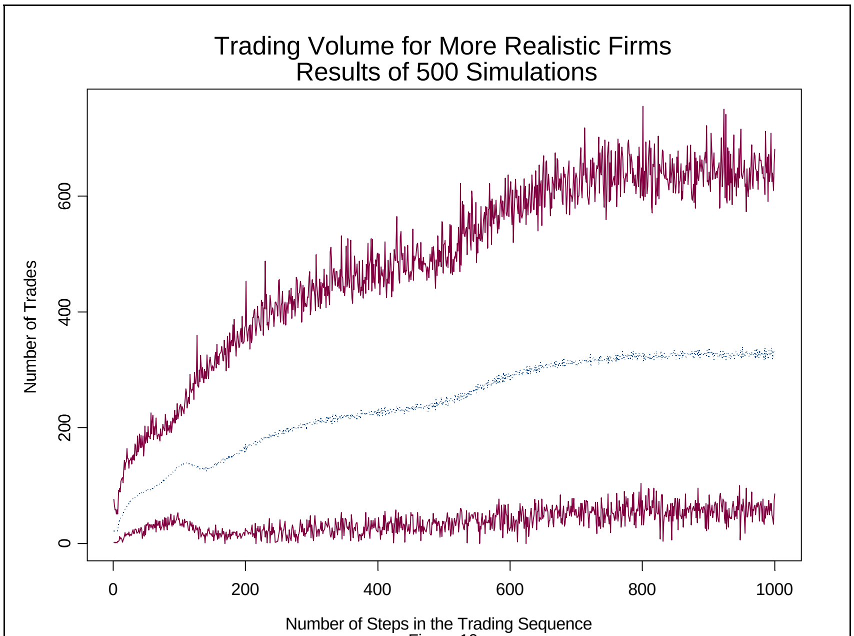

Figure 10

Figure 11 shows the results when the number of permits is allocated in proportion to the sizes of firms, plus the larger error term. This means that for 95 percent of firms, the number of permits allocated is plus or minus 40 per cent of they would have been allocated just on the basis of relative size,. Perhaps surprisingly, this does not make a great deal of difference to the overall number of trades carried out, with the number per period settling at just under 400.

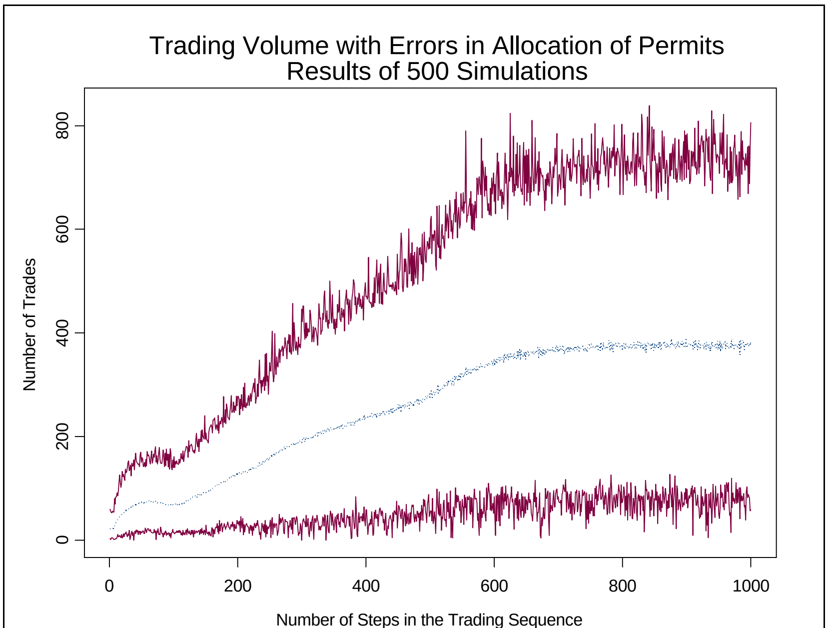

Figure 11

Finally, Figures 12 and 13 show the level of market price with 271 heterogenous firms. Figure 12 reports results with permits allocated exactly in proportion with output, and Fgure 13 shows them allocated with error as in Figure 11. The middle, dotted line is again the average price of 500 repeated solutions of the model.

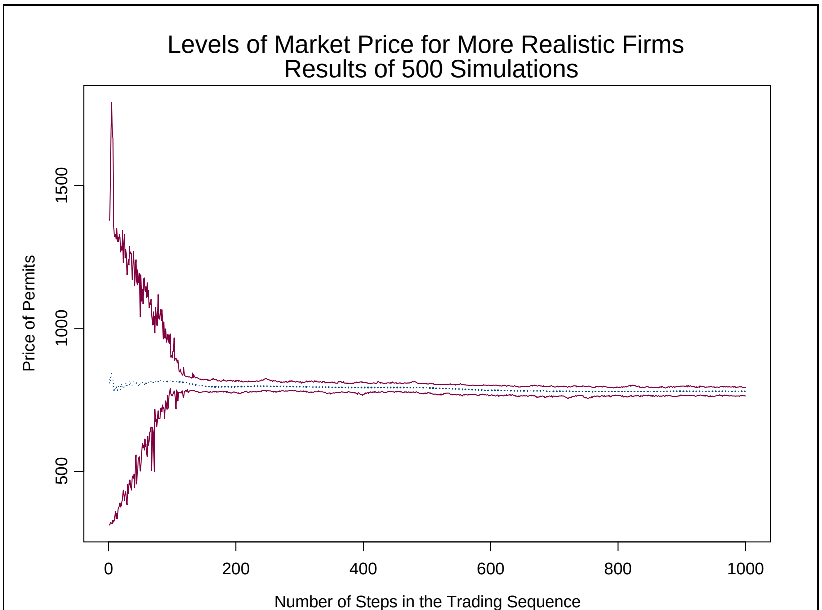

Figure 12

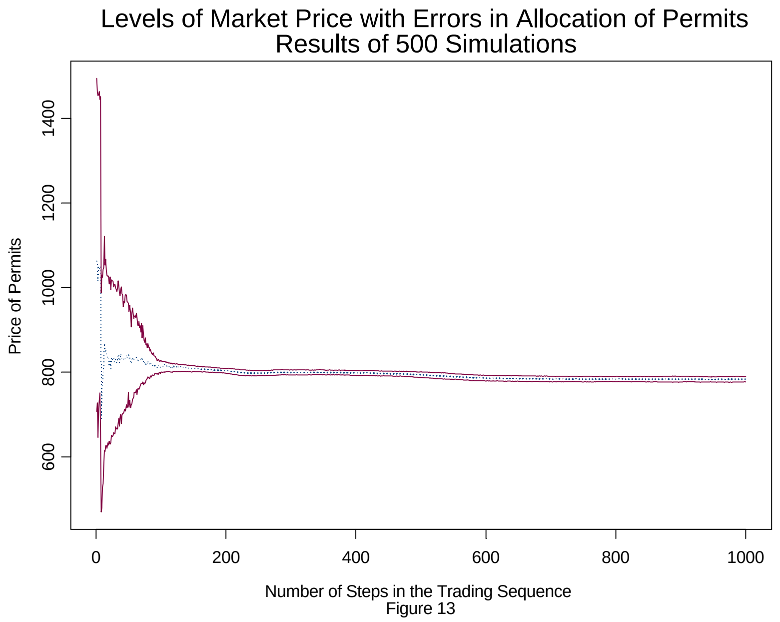

Figure 13

In summary, the results of this section re-inforce those of section 3.1. A market in emissions permits is likely to be orderly, in the sense that a reasonably large number of permits will be traded and, after an initial period of learning about the value of permits, the price of permits does not in general exhibit large fluctuations.

3 Conclusion

This simple market model enables us to study the behaviour of firms who have no experience of the market for permits and who use very simple rules to modify their bids and offers. In general, despite the fact that firms are deliberately given naive rules of learning about the value of permits, and that by construction the level of trading is thin, the price of permits converges rapidly around its theoretical equilibrium level. In the context of the model, the values of both output and pollution are at their optimal levels.

Even with a very small number of firms who, by assumption, have little incentive to trade, a market does emerge which is orderly. The price converges to around its equilibrium level, even though the number of trades actually carried out is small.

The greater the degree of difference between firms, in terms of their sizes, the degree at which they pollute, and the costs they incur in abating pollution, the higher the number of trades which will be carried out in the permits market.

This model, despite its simplicity, provides a simple and flexible tool for testing a number of interesting hypotheses about the evolution of prices on such a market and on the production and pollution levels attained. Many questions of interest to the public authorities involved in this area can be examined in extensions of the model.

Appendix 1:
Mathematical Statement of the Model

We assume that the firms are identical in the structure of their costs and revenue. They only differ in terms of parameter values (α, β, γ). The cost of producing y units of output is given by

$$C(y) = y^2 / \gamma$$

and the revenue from selling y units of output is

$$R(y) = \rho\, y.$$

In the simulations carried out we set $\rho = 1000$. The costs of production increase quadratically but the revenue generated rises more slowly.

Each unit of production is assumed to generate α units of pollution, i.e. if P(y) is the level of pollution associated with the production of y units of output then

$$P(y) = \alpha\, y.$$

The parameter α was given the value 0.5.

The cost of abatement was also taken to increase quadratically, i.e. the cost of reducing the level of pollution by z units is

$$A(z) = (\beta / \gamma)\, z^2.$$

If a firm is able to produce z_0 units of pollution as a result of holding the necessary permits, then the cost of reducing its level of pollution to z_0 when producing y units of output is given by

$$A_0(y) = (\beta / \gamma) (\alpha y - z_0)^2$$

(This assumes that the amount of pollution arising from this level of output is greater than z_0.)

Combining the costs of production and abatement, we can find the marginal cost of producing a unit of output,

$$MC(y) = 2 y / \gamma \qquad \text{if } \alpha y < z_0$$

and

$$MC(y) = 2 y / \gamma + 2\alpha (\beta / \gamma) (\alpha y - z_0) \qquad \text{if } \alpha y \geq z_0.$$

In the first case the level of pollution is below the permitted level z_0, in the second case it would otherwise be above z_0 and so incurs an abatement cost.

The level of output of a firm, given a certain number of permits can then be calculated by equating the marginal revenue and marginal cost curves and solving for y.

Similarly, if the firm has a number of permits, we can work out the price that it will be willing to pay for an extra permit. This will be just equal to the amount of extra profit that will be gained from the extra output produced as a result of having the extra permit. Explicit details can be obtained from the authors.

Homogeneous Firm Examples

The data in the report on homogeneous firms was generated with identical costs and revenues for each firm. The parameter values were:

$\alpha = 0.5$, $\beta = 15$, $\gamma = 1$, $\rho = 1000$

and each firm was given 100 permits. These parameters mean the firms should value an extra permit at 944 (which we can see that they do from the results in Figures 1,3,5 and 7).

Appendix 2:
Calculations for Estimating Parameters for more Realistic Firms

The left hand side of the table below shows data taken from the DTI: 'Small and Medium Enterprise (SME), Statistics for the United Kingdom, 1998'. Together with data from the Marshall Report: 'Economic instruments and the business use of energy', November 1998. The right hand side shows the estimates used in our model of more realistic firms (explanation below).

	Actual Data			Estimates for Model		
SIC Code	Number of firms	Gross output £m	Est. CO_2 emissions (000s of tonnes)	Number of Firms	Estimate of alpha*	Estimate of gamma**
Mining & quarrying	1270	3566	588	1	1.65	2.81
Food & Drink	7320	50704	4376	7	0.86	6.93
Tobacco	25	8655	60	0	0.07	346.20
Textiles	5580	10514	904	6	0.86	1.88
Clothing	6930	5757	178	7	0.31	0.83
Leather	1225	2035	69	1	0.34	1.66
Timber	7215	4928	465	7	0.94	0.68
Pulp & Paper	2585	3997	1923	3	4.81	1.55
Printing	22950	35393	680	23	0.19	1.54
Chemicals	3920	42074	7196	4	1.71	10.73
Rubber & Plastics	6800	17771	2342	7	1.32	2.61
Cement etc	5060	10973	2597	5	2.37	2.17
Basic metals	2450	18258	9170	2	5.02	7.45
Metal products	24960	23453	673	25	0.29	0.94
Machinery	14015	31428	1274	14	0.41	2.24
Electrical machinery	5220	12140	397	5	0.33	2.33
Radio & Television	2735	14205	353	3	0.25	5.19
Instruments	5920	9869	125	6	0.13	1.67
Motor vehicles	3140	31970	1053	3	0.33	10.18
Other Transport	2920	14181	663	3	0.47	4.86
Furniture	14480	9973	716	14	0.72	0.69
Construction	124680	90178	1163	125	0.13	0.72
TOTAL	271400	452022	36965	271		

* Pollution per unit of output (tonnes of CO_2 per £00s of output)
** Average size of firm £mn

Number of Firms in the model was calculated by dividing the actual number of firms by one thousand and rounding to the nearest integer, therefore it is as if one in every thousand firms decides to trade.. For each classification sector, alpha was calculated by dividing estimated CO_2 emissions in 10,000s of tonnes by Gross output in millions of tonnes, then multiplying by ten. This gives an average amount of pollution per unit of output. Gamma was calculated by dividing Gross output by the actual number of firms, to give an average size of firm in each classification sector. No data is currently available on the cost to firms of abating pollution. Thus we estimated the beta value for each firm by drawing it from a lognormal distribution with mean 15 and standard deviation 0.7, giving a minimum of 3 and a maximum of 80.